Simulation Model for Estimating Risk of Uncertainty on Return oN Investments OF Public Investments

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ANALYZING an investment is usually not a difficult problem, based on the dynamic indicators of return of investments (i.e. NPV, IRR). However, it is not so simple if the aim of these investments is to prevent damage to economic objects, agricultural areas, cultural sites, which

may be inflicted by unpredictable natural phenomena (i.e. floods). Since conventional methods are not practical in this case, a stochastic simulation model was developed based on the Monte Carlo method to estimate the risk of the return of these types of investments.

Stochaistic simulation models have proved that the issue of economic profitability can be raised in the case of public investments as well. The expected benefits of the investment determine whether it is justifiable to invest into blocking out negative impacts or other strategies are necessary to decrease their destructive effects. The research was supported by the Hungarian Scientific Research Fund (OTKA) No. K63231 research project.

Key Words: Uncertainty, Risk, Simulation Model, Externalities, Public Investment.

Introduction

According to Samuelson and Nordhaus (1985), public goods have in common the sense that each individual's consumption of such a good leads to no subtractions from any other individual's consumption of that good.

Private enterprises and the government hold different stakes in private and public goods and property, depending on the size and efficiency of the state as well. Pure public goods are provided and maintained by the state or government (Kaul and Mendoza, 2006). The question arises, can the establishment of public goods be examined and justified from the point of view of economic efficiency and profitability.

Traditional investment efficiency analysis methods are not suitable for examining the return on public investments, because the income and benefits generated by the existence of the public goods cannot be quantified (Kaul et al. 2003).

On the other hand, cost-benefit analysis differs from direct financial approaches. It is a quantitative decision-making method to assess investments, which considers all benefits and costs or damage irrespective of where or how they occur. Cost-benefit analysis expresses costs and benefits of alternatives in financial terms, and the comparison of alternatives provides insights into optimal decision-making. The present paper is not going to discuss theories on the evaluation of public goods; instead, a specific type of public goods, flood control systems are selected for detailed analysis on the economic justifiability of establishing public goods by extending the conventional dynamic return of investment analysis methods.

First of all, a few facts to underline the importance of flood dikes and levees as public goods: hurricane Katrina hit the New Orleans area in the United States in August 2005, and caused the death of hundreds, destroyed tens of thousands of homes, businesses and factories, and it took months to remove water from the affected areas. The estimated damage reaches billions of dollars.

The importance of the issue of flood control is not to be underestimated in Hungary either; regarding the areas prone to flooding, the following economic and demographic data were collected in a survey in 1999 (Szlávik, 1999):

- \bullet 2.5 million people (25% of the total population) living in almost 700 settlements are affected;
- \bullet one third of the agricultural crop land in Hungary (1.8 million hectares) are in flood areas, the production value amounts to 2.8 billion EUR at present prices;
- •32% of the railroads and 15% of the public roads, worth 3.7 billion EUR, run through flood affected areas;
- \bullet more than 2000 industrial plants are situated in these areas, with a property value of 7.5 billion EUR and producing 16 billion EUR worth of goods per year;
- **v**in all economic sectors, the annual gross production amounts to 21 billion, which constitutes almost 25% of the gross national production;
- the value of the accumulated national wealth amounts to 33 billion EUR.

In addition to the economic, natural and cultural values, which accumulated during the past centuries, the value of human life is also significant, though can hardly be quantified in financial terms.

Flood control systems are public goods which generally fail to bring direct economic benefit and profit, and the rate of directly attainable benefits falls significantly short of the indirect benefit. The following alternatives can be considered from the point of view of the economic impact:

1. Flood control investments are unnecessary, because the entire flood area cannot be utilized economically; it is under the influence of natural forces.

 Input=0, but the benefit from economic utilization (annuity) also drops. This alternative can be applied in case of abundant resources (e.g., land area).

- 2. People live together with the effects of natural forces in the flood area and accordingly, housing, economic establishments and utilities are constructed using technological solutions that withstand the floods; consequently, floods cannot cause significant damage. Application of the alternative is associated with important supplementary inputs.
- 3. Economic establishments in the flood areas are protected by flood control systems, construction and maintenance require significant inputs, but the benefits are clear in the reduction of the damage value or lack of damage. Construction and maintenance costs of establishments in the protected areas are lower than it would be in the case of establishments that need to withstand the floods. The costs of construction and maintenance of the flood systems are lower than the savings from the above establishments and the losses suffered in case of floods combined.
- 4. Combination of alternatives 2 and 3.

In economic sense, the selection of the suitable alternative may be carried out on the basis of maximum direct and indirect benefits, attainable by means of the variables involved.

In constructing the simulation model, a number of limiting conditions had to be taken into account. Among them, the following ones must be emphasized:

 \bullet Economic changes behave like discrete values (Fig. 1), they are not be described by continuous functions with n variables, i.e. the problem cannot be solved by function analysis;

Figure 1: Satellite Image of Intensively Utilized Riverbanks, with Endangered Economic Values Matrix

Source: Google Earth with own Additional Construction viewed on 2009.

 The area in question can be subdivided into regular smaller areas (Fig.1) within which the variables (such as expected water height, economic value intensity, number of days with water cover, etc.) are evenly distributed. As a result, the total area can be described as a matrix of k×l elements, with average (e.g. water height) or accumulated, aggregate quantities (e.g. value of establishments).

- The variables are of stochastic character, the location of the goods may change in space, their value is changing in time, the change is random;
- Collection of data would require disproportionately high costs that would not be justified by the obtainable results;
- Application of the finite element method may emerge, but because of the reasons outlined above, the use of the method can be rejected partly due to data generation reason (totality of details, reliability, ignorance of function realizations, etc.) partly due to its cost/benefit ratio;
- \bullet With the help of simplifying conditions, the estimation algorithm may be simplified, supposing quasi uniform distribution of several variables; in other cases, supposing a characteristically linear relationship among functions with m variants $(m<\n< n)$, description of the development of functions in time and in space is replaceable.

Material and Methods

The model examination was carried out using data on four sections of the river Tisza, having important economic potential. Based on the real data, six different economic wealth intensity level (non, weak, average, intensive, very intensive and super intensive economic activities) were generated. The exact parameters of these categories will be identified later on.

The basic model for the investment efficiency investigations was provided by the traditional NPV and IRR indices [Gittinger 1982], with the difference that the cash flow structure was as follows:

Benefits in the tth year (+):

+ direct incomes from utilization $(B_{i,t}^d)$

Figure 2: Matrix of the Terrain

Source: Own Construction.

Figure 3: Spatial Arrangement of Economic Values

Source: Own Construction.

traditional utilization (grass yield) and alternative utilization (energy forest, beef cattle keeping)

+ indirect (quasi) benefits (value of the lack of damage) $(Bⁱ_{j,t})$

in human life; in factories, buildings, establishments; agricultural production; in natural values; in cultural values

Investment costs in the tth year (-):

- costs of dike-body construction or raising $(\mathbf{I}_{k,t}^b)$ and
- costs of setting dike surface covering $(I_{l,t}^c)$

Continuously emerging costs in the tth year $(C_{m,t})$ (-):

- maintenance costs (mowing, etc.)
- costs of setting and demolition of temporary flood control instruments
- reparation costs of damages in flood control works
- technological costs of alternative utilization methods
- costs of elimination of damage inflicted on the protected areas

Cash flow in the tth year (CF_t), i.e.

$$
CF_{t} = \sum_{i=1}^{m} B_{i,t}^{d} + \sum_{j=1}^{n} B_{j,t}^{i} - \sum_{k=1}^{o} I_{k,t}^{b} - \sum_{l=1}^{p} I_{l,t}^{c} - \sum_{h=1}^{q} C_{h,t}
$$

Requirement for the net present value of the cash flow is

$$
NPV(B, I, C, t, r, p) = \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t} \rightarrow
$$

where:

 $r =$ alternative interest rate $(\%)$

 $T =$ analyzed period (T> Δt) (year); in the simulation T was 15 years

 $p =$ risk of flood $(\%)$

The system is formed upon the effects of countless, random events. Year by year, changing water outputs and floods of different durations must be taken into account, their direct and indirect effects have a significant effect on the economic judgment of the flood control system. The Monte Carlo simulation is a suitable method for this examination; with its help, all possible combinations may be examined and a comprehensive study on the whole distribution of the project outcomes is possible [Brealey and Myers 1991].

The aim of the simulation model constructed on the basis of the above aspects is to take into consideration both the direct benefits attainable in areas confined by the flood controlling earthdikes and the indirect benefits such as the prevented damage in human, natural and material wealth on the protected side. An additional aim if to analyze the changes occurring upon the effect of the system variables and to conduct studies on investment efficiency of the system, by taking into account the investment and the maintenance costs necessary to attain these benefits. Basic relations in the model are presented in Figure 4.

The analysis was made with the help of Monte Carlo simulation, based on a simulation technique capable of dealing with generating a large number of random values.

- 1. The steps of drawing up the mathematical method:
	- defining the phenomenon and the input variables,
	- drawing up the equation of the model $Y = G(X_1, X_2, \ldots, X_M)$,
	- assigning probability distributions to the input variables X_1, X_2, \ldots, X_M
		- o selecting distribution graphs
		- o defining values for the parameters of the graphs.
- 2. generating N random values for each Xi input variable according to the distribution assigned to Xi,
- 3. evaluating the equation of the model in N points,
- 4. Calculating uncertainty range from the N values of Y quantity, either directly from the statistics of the Y values or by drawing up the approximate continuous distribution graph.

max

Figure 4: Cost-benefit Model of the Flood Controlling Earth-dike

Source: Own Construction.

The simulation was conducted using N=100.

Areas of varying economic intensity alternate along riverbanks. The flood basins, which are units of areas from the point of view of flood defense, were classified into six categories based on their natural and economic characteristics, ranging from non-active (natural) areas to very intensive (metropolitan) areas. The parameters for describing the natural and economic characteristics of areas used for modeling in the simulation models are summarized in Table 1.

Due to global climate change, average temperatures have been rising in the Carpathian Basin, whereas total rainfall is decreasing; at the same time, however, the irregular pattern of the distribution of precipitation increases the unpredictable fluctuation of water levels and the maximum flood levels are alarmingly on the rise.

The following assumptions are made:

- 1. On the protected area, the spatial distribution of the economic values is even, i.e., ?a (value intensity, currency unit/ km^2) is uniform for the given flood basin.
- 2. The value increases as time passes, the change is linear, and the average rate of increase in the area equals the average national rate of economic growth.

 $\Phi_{a,t} = (1 + \varphi) \cdot t + \Phi_{a,0}$

where:

 $\Phi_{\alpha t}$ = value intensity of flood basin "a" in year "t" (billion currency unit/km²)

 φ = average increase of the indicator characterizing economic growth (GDP, in %)

 Φ_0 = value intensity of flood basin "a" in previous year (billion CU/km²)

3. The days of water cover are dependent on river level and are characterized by gamma distribution.

$$
n = N \cdot \tau \left(\frac{h}{H}, \delta, \alpha, \beta \right)
$$

Economic activities	Characteristics	coefficient	Value intensity Population density coefficient
none	Not inhabited, only used for general agricultural purposes	Ω	0
weak	Sparsely populated, used for general agricultural purposes with low intensity	0.5	0.5
average	Average population, used for general agricultural purposes with average economic intensity	1	1
intensive	Densely populated, used for agricultural purposes, above average intensity	1.5	3
very intensive	Regional economic center, densely populated, used for intensive economic activity	$\overline{2}$	6
	super intensive National economic, industrial center, urban environment, densely populated, very intensely utilized area	3	10

Table 1: General Characteristics of the Classified Areas used in the Simulation Model

Source: Own Classification.

where:

n = days of water cover associated with h water level;

 $N =$ number of days in the year;

$$
\tau\left(\frac{h}{H}, \delta, \alpha, \beta\right) = \text{gamma}
$$

distribution value, dependent on relative water level (h/H), transformational coefficient (δ), parameters of the gamma distribution graph (α,β).

One of the results of the research is the simulation model itself. One group of the variables in the model (direct incomes, technological costs of utilization) may be regarded as quasi constant, while indirect benefits and associated costs resulting in risk change (investment costs, costs of constructing and demolishing temporary flood control establishments) and the reconditioning costs of the losses inflicted on the protected areas, respectively, are represented as a function of the risk in the simulation model.

The following stipulations were made:

- Cash flows are modeled at unchanged price on the price level.
- In the simulation model, temporal changes are represented by the trend functions of the changes, ensuring in the natural and economic environment.
- The value of the variables substituted in the course of the modeling is the value of the trend function at the ith point of time plus a random value in the $\pm 2\sigma$ range:

 $p'(t) = p(t) + 2\sigma(2\rho - 1)$

where:

 $p'(t)$ = the substitution parameter, belonging to the ith point of time;

 $p(t)$ = the function value of the trend function at the ith point of time;

σ= standard deviation of the trend function; random number in [0;1] range.

Results

A crucial stage in the Monte Carlo simulation is selection of the distribution type of the graphs. In the case of flood protection systems, the key distribution pattern is the distribution of water levels. For the current analysis, the research relied on data collected at Tiszaug water level measuring station, and arrived at several important conclusions. Looking back on the past 16 years, it can be ascertained that the yearly fluctuation of water levels is significant (Figure 5); a pattern of spring and autumn peaks emerges from the graph constructed on minimums, maximums and average; however, occasional summer peaks can be observed as well.

The large variability of water levels raises the question of overall trends in extremes (peaks and lowest water levels). The graph of yearly maximum and minimum water levels reveals that the expected highest water levels are on the rise (average rate of increase: 17.4 cm), but there is also a wide fluctuation in maximum water levels (30%). The rate of increase in the lowest water level is slower (4 cm per year), and the fluctuation is also smaller (19%). The different rate of the changes in the extremes means that the gap between maximum and minimum water levels is widening.

The frequency of individual water levels (i.e., how many days per year each numerical value was measured) is characterized by gamma distribution, which is the most appropriate to describe the change in average values. Decreasing the scale decreases the variability of the distribution; however, the characteristic pattern of the distribution does not change, only the α and β values of the gamma graph. The parameters of the gamma graph were determined by heuristic iteration.

Figure 5: Water levels at Tiszaug, on the Tisza River, 1990-2005 *Source: Own Calculation.*

Transforming the gamma distribution graph placed on the average water levels by $\pm 2\sigma$, with the restriction that water level frequencies may not be negative numbers, the minimum and maximum values of the given occurrences are expected to fall in the given range. (Table 2, Figure 6.)

Scale of water level		Parameters of the gamma graph	Correlation coefficient		
	α	β	Graph of average and Γ	Graph of minimums and Γ - 2σ	Graph of maximums and $\Gamma\Box + 2\sigma$
m	4.015	0.550	0.89030	0.95382	0.90557
m	4.000	1.000	0.59069	0.57109	0.82006
dm	2.200	15.000	0.97512	0.68983	0.93707

Table 2: The Correlation of Approximate Gamma Graph (Γ**) and Average, Minimum and Maximum Water Levels**

Source: Own Calculation.

Figure 6: Gamma Distribution ± **2**σ **Transformed Gamma Distribution on Relative Frequencies of Water Levels, at Tiszaug, Tisza River, 1990-2005**

Figure 7: Gamma Distribution of Duration of Water Levels (in days)

Source: Own Contruction.

One of the problems arising during the simulation process was the quantification of the damage to agricultural areas, since it depends on the characteristics of agricultural use in the given area (crops, average yield, etc.), and on the duration of flood cover. Based on the specific data available for the examined four flood basins, linear approximation is appropriate to estimate damage to agricultural areas (Figure 8). Table 3 summarizes the parameters of the graphs.

Basin	$\alpha_{\rm a}$	٣а	\mathbf{R}^2	
А	0.0063	0.0213	0.7738	
B	0.0533	-0.0637	0.9997	
$\mathbf C$	0.0256	-0.0313	0.9987	
	0.0139	-0.0043	0.9975	

Table 3: Coefficients of Agricultural Damage Graph (billion CU)

Source: Own Calculation Based on Halcrow, 1999.

The graph for agricultural damage is the following:

$$
\Psi^{\,a}_{a,t} = L_{a} \cdot \frac{2 \cdot H}{L^{v}_{a} \cdot h} \cdot \left(\alpha_{a} \cdot n + \beta_{a} \right)
$$

where:

H = highest recorded maximum water level (cm)

 L_{a}^{v} = length of flood defense system in the flood basin (km)

h = current water level (cm)

$$
B_{a,t}^i = \Psi_{a,t}^x
$$

where:

 h_{max} = highest expected water level for year "t" (cm)

$$
h_{\max, t} = \overline{h}_t + 2\sigma_h (2\rho_h - 1)
$$

$$
\overline{\mathbf{h}}_{t} = \mathbf{a}_{h} \cdot \mathbf{t} + \overline{\mathbf{h}}_{b}
$$

 $=$ expected mean of average water level in year "t" (cm)

- = expected mean of average water level in base year (cm)
- a_h = expected annual rate of increase in the mean (cm/year)

Average number of days with flood cover:

$$
\overline{n}_a=N\cdot \underset{h_0}{\overset{h_{max}}\int}n_{a,t}dh
$$

where:

= average number of days with flood cover in basin "a" (days) $N =$ number of days in the year)

$$
\int\limits_{h_0}^{h_{max}}\!\!n_{a,t}dh
$$

distribution graph of water level Number of days with flood cover:

$$
n_{a,t}=\overline{n}_a+2\sigma_a^n(2\rho-1)
$$

Source: Own Construction

The estimation of indirect benefits of flood control requires special approach. The starting assumption is that benefits equal the expected cost of reconstruction and repair of damage in human, economic, natural and cultural values in the area that would be inflicted in the absence of flood control systems. The extent of the damage depends on:

= gamma

- zThe value of all human lives (number of inhabitants), the value of economic, natural and cultural goods, which are changing in real value during the investigation period;
- \bullet Distribution of values in the protected area;
- \bullet The height of the flood;
- \bullet The topography of the protected area or the pattern of the elevation in the area;
- •The height of water between the dikes;
- \bullet The size of the area covered by the water mass of the flood;
- •The duration of the flood;
- •The probability of the occurrence of flood.

Variable			
Name	Symbol	Unit	Value
Income from conventional farming (grass yield)	$B^{(f\tilde{u})}$	M CU/km	0.5
Income from alternative farming (cattle)	$B^{(hm)}$	M CU/km	0.5
Income from alternative farming (biofuel plants)	B ^(en)	M CU/km	$\mathbf{1}$
Base value intensity of property at risk	$\overline{\Phi^V}$	M CU/km ²	2500
Average distance between dike maximums	1	m	500
Average increase in elevation of the terrain	γ	degree	2
Overflow coefficient	M	m	14336
Overflow water level	h_0	cm	100
Gamma distribution of water levels, parameter α	α		$\overline{4}$
Gamma distribution of water levels, parameter β	β		$\mathbf{1}$
Average overflow level	h _{átl}	cm	600
Standard deviation of average overflow level	Ω	cm	100
Water level on water gauge	h	cm	756
Area threatened by flood	L	km^2/km	0.306731
Risk of flood on area without protection	μ r	10-6person/pers.	42.7
Population density	$\mathbf D$	person/km ²	250
Average value of human life	p^E	M CU/person	20
Value intensity of human life	$\overline{\Phi^{\rm E}}$	M CU/km ²	0.2135
Value of human life at risk)	Ψ _e	M CU/km	0.065487
Average rate of economic growth	φ	$\frac{0}{0}$	$\overline{4}$
Base value intensity	Φ_0	M CU/km ²	2500
Probability of flooding (overflow)	\mathbf{p}		$\rm 0.95$
Highest recorded water level	\overline{H}	cm	750
Average number of days with flood cover in the basin	$n_{\text{átl}}$	day	21.58809
Standard deviation of average number of days with	Ω	day	$\bf 5$
flood cover			
Coefficient for typical agricultural damage in basin	a	M CU	53.3
Coefficient for typical agricultural damage in basin	b	M CU	-63.7
Total length of flood protection in basin	L_{v}	km	74.2
Average rate of water level increase	dh	cm/year	$\overline{5}$
Intensity of natural values	B_{N}	M CU/km ²	100
Intensity of cultural values	B_C	M CU/km ²	500
Cost of constructing dike	I^g	M CU/cm/km	$\overline{5}$
Value of construction of dike	I^g_{0}	M CU/cm/km	4250
Maintenance	\mathcal{C}	M CU/km	10
Average total cost of flood defense	$\overline{C^{v\acute{e}d}}$	M CU/day	10
Base cost of flood defense	C_0 ^{véd}	M CU	10

Table 5: Starting Parameters for the Simulation, Average Values, per Km

Source: Own Construction.

Figure 9: Distribution of Indicator of the Investment-efficiency Model of Flood Control Investment (NPV) Values According to the Monte Carlo Simulation

Source: Own Construction.

Table 6: Estimated Risk of Return of Investment into Public Goods According to Monte Carlo Simulation

Economic activities	Estimated Risk of Return of Investment	
non	97%	
weak	89%	
average	73%	
intensive	50%	
very intensive	35%	
super intensive	7%	

Source: Own Calculation.

As a part of global climate changes, average temperature is rising in the Carpathian Basin, the amount of annual precipitation is decreasing; at the same time, however, fluctuation of the water level is increasing due to the changes in the distribution of the precipitation. The maximum water levels show an increasing tendency as well, which requires further expansion of the protection capacity.

Calculation of benefits applied in the model in the course of investment-efficiency analysis differs from the usually used ones, and will be presented in Table 4.

The present study does not describe the relevant mathematical relationships in detail due to the constraints of the publication; they were calculated and determined in line with conventional methods used in cash flow models.

The simulation was conducted as described in the Materials and Methods section. Table 5 summarizes the starting parameters, based on national average values. Results are summarized in Figure 9.

Conclusion

According to the results of the simulation, the likelihood of investment return of the public goods investment is below 50% in the case of non, weak or average economic intensity, i.e., the risk is high. The probability of investment return exceeds 50% in the case of densely populated, intensively utilized areas (regular agricultural or industrial production), and is thus economically advisable to invest into public goods, i.e. flood defense systems, as an alternative to individual or occasional reconstruction and damage reduction methods (Table 6).

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