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INCORPORATION OF LEARNING CURVES IN BREAK-EVEN POINT ANALYSIS

Krishan Rana
Suneel Maheshwari
Ramchandra Akkihal

THIS study illustrates that a break-even point may change with an improvement in the learning curve. Traditionally, break-even point (BEP) is assumed to be a unique value. Implications of this contradiction to the traditional concept of a unique BEP are significant. We feel that introduction of learning curve to BEP may have significant applications in areas of production planning, investing, financing, bidding for contracts, and other areas of business operations. The relationship between a learning rate and the BEP may also change how the concept of BEP is taught in the areas of economics, production/operations research and accounting.

Introduction

Management makes many critical decisions that affect a firm's profitability and is often interested in knowing the impact of a certain decision on the firm's profitability. For example, it is not unusual for managers to be excited about an installation of a new, more efficient plant based on an optimistic sales forecasts and promise of a larger market share. However, it is necessary to understand how this decision will affect the firm's profitability. Cost-volume-profit (CVP) analysis can help managers assess the impact of their decisions on profitability.

CVP analysis involves studying the interrelationships between costs, revenues, level of activity, and profits. Thus CVP can answer questions like what happens to the profitability if the fixed costs increase due to expansion, or how many units need to be sold before the firm can cover its cost and profit from expansion. The point at which the total cost equal total revenue is called the break-even point (BEP). BEP can be expressed in terms of numbers of units to be sold or dollar value of sales. BEP in terms of quantity (units) is defined as fixed cost divided by the contribution margin per unit, which is defined as the selling price per unit less the variable cost per unit. It is assumed that the variable cost per unit remains constant and total fixed costs do not change in total for a relevant range of production.

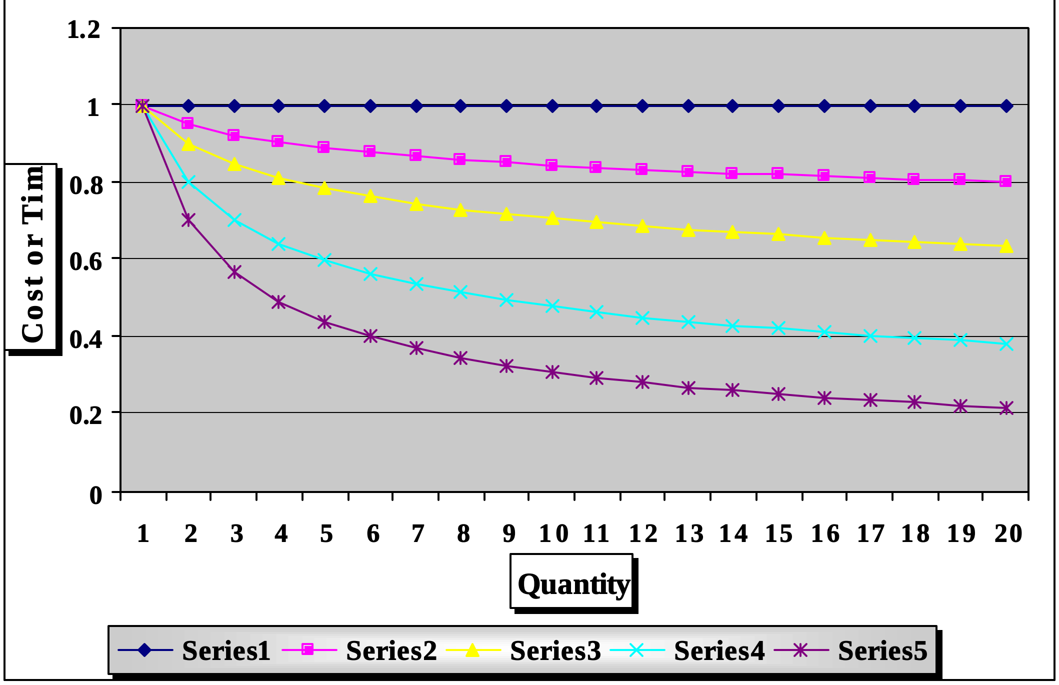
BEP can be useful in assessing the risk of selling a new product, setting sales goals and commission rates, planning for the manpower requirement, deciding on marketing and advertising strategies, planning for production capacity, and other similar operating decisions. BEP analysis is an important theoretical concept and is included in most production/operations management and accounting textbooks.

The concept of BEP, as perceived today, does not allude to the possibility that BEP may not be a unique number. We believe that the BEP is dynamic and will vary over time. As the corporations become more familiar with the production process they will invariably become more efficient in their use of organizational resources. Consequently, lesser resources, material, labor hours or other costs will be consumed for every unit produced and sold. This process of continuous improvement and efficient use of resources can be shown through learning curves.

Learning Curves

According to one of the definitions, learning curves may be defined as the plot of the cost level against cumulative output (Petrakis, Rasmusen, and Roy 1997). Learning rate refer to the improvement in the performance of activities performed repetitively. As operators of a firm gain experience with a product they make improvements to the product and process which produces the product. With proper instruction and repetition, workers learn to perform their jobs more efficiently and effectively. A learning curve or improvement curve is a graph that reflects the relationship between increasing repetitions and a decreasing time per repetition. Consequently, the direct labor hours per unit of a product are reduced. Since labor hours required for unit production decrease, so does the variable cost of production. Similar affects may be there on other elements of costs. This process of reduction in cost with different learning curves is graphically represented in Figure 1.

Figure 1
An Example of Learning Curves



We will take a generic example and compute BEP for different learning curves. Economists have been aware that a firm's cost curve for producing a given item may shift down over time as learning occurs (Spence 1981; Majd and Pindyck 1989). Theory of learning curves states that as the cumulative number of units produced is doubled, cost per unit is reduced by the learning rate. For example, suppose it takes $1.00 to produce the eighth unit and if the learning effect is 90% it will cost $0.90 to produce the 16th unit. Also, the cost of producing between 9th and 15th unit will be between $1.00 and $0.90. In the learning curve terminology a learning rate of 70% is considered more effective than 90%. Thus if we use the learning effect, the variable cost to produce will vary with the production and consequently the BEP. Thus we contend that accountants' assumption that variable cost is constant is not true because of the learning effect. Therefore, the cost of the first unit will be higher than the subsequent unit when learning effect is considered.

Following paragraphs define BEP and learning curves.

Then for a break-even quantity, say Q_{BEP}

Q_{BEP} = FC/ (R - VC) (1)

Let

FC = Fixed cost

VC = Variable cost per unit

R = Revenue per unit or the selling price

Q = Quantity

Then Profit = Total Revenue - Total Cost

Or Profit = Q (R - C) – FC (2)

For a given BEP cost of nth unit using the definition of learning curve is given by

$Y_n = S n^b$ (3)

Where

Y_n = cost of the nth unit

S = cost of the first unit

n = cumulative number of units produced

b = Ln r /Ln 2

r = the learning rate

(For example, 80%(.8) learning is faster than 90%(.9) learning.)

To compute the cost of any n units we can use the following formula

$S_n = ¿_{k=1}^{n} S k^b$

Or $S_n = S ¿_{k=1}^{n} k^b$ (4)

Where

S_n = total cost of n units

b = Ln r /Ln 2

r = the learning rate

In terminology of learning curves, an 88% learning curve gives b= Ln(.8)/Ln(2) = -0.3219. The value of b is always a fraction between -1 and 0. The right hand side of equation (4) is a series, and mathematically, its solution does not exist in a closed form. In order to obtain its solution, we approximate k^b as follows:

$$k^b = \int_{K-0.5}^{k+0.5} k^b \, dk \tag{5}$$

Also, we know that $1^b = 1$ and $2^b = r$, the learning rate, then

$$S_n = S\left(1 + r + \int_{3-0.5}^{n+.5} k^b \, dk\right) \tag{6}$$

$$\text{Or } S_n = S\left(1 + r + \frac{(n + 0.5)^{b+1} - 2.5^{b+1}}{b + 1}\right) \tag{7}$$

Here, equation (7) is the total variable cost of producing the first n items (cumulative). If n is the break-even quantity, then

$$F\,C + S_n = n\,R \tag{8}$$

Equation (8) does not yield a closed form solution for n. Therefore, the value of n can be determined by trial and error using a spreadsheet. Nevertheless, if fixed cost, revenue per unit, and the variable cost of the first unit is known, determining n is a simple task. Table 1 demonstrates the comparison of BEP and profitability at varying rates of learning curves. We assume the following values:

Fixed Cost (FC)	=	$200,000
Selling Price (R)	=	$50
Variable cost of the first unit	=	$30

If no learning is assumed, then BEP is simply 10,000 units. Table 1 shows the effect of improvement in learning rate on the BEP and profitability. The learning rate is assumed to improve in increments of 2.5%. Table 1 shows that as the learning curve improves (goes from 100% to 70%) variable cost per unit goes down. At 97.5% learning the variable cost is $22.50 per unit and it goes down to $5.45 per unit for 85% learning. Personal computers are a good example of where the cost of the product might have been significantly influenced by the learning curves. The actual learning may or may not take place at this rate.

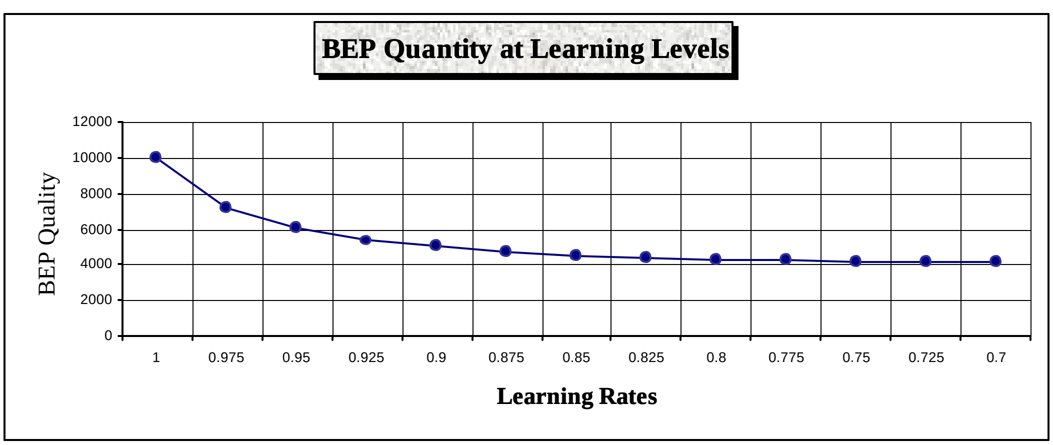
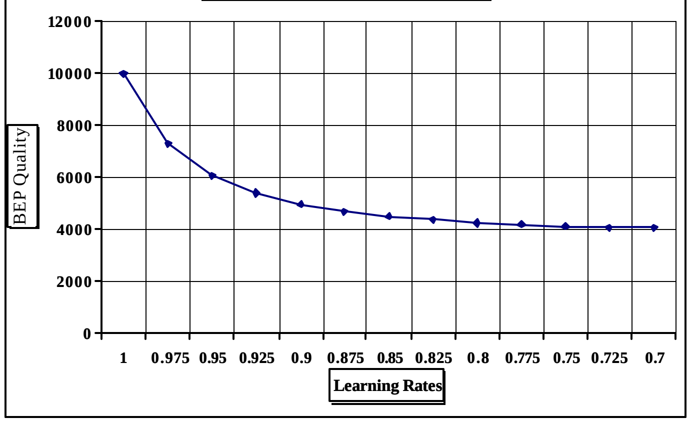


Table 1: Computation of quantity at break-even point for different levels of learning

	100%	97.50%	95%	92.5%	90%	87.5%	85%	82.5%	80%	77.5%	75%	72.5%	70%
Learning Rate (r)	1	0.975	0.95	0.925	0.9	0.875	0.85	0.825	0.8	0.78	0.75	0.73	0.7
Assumptions b = ln r/ln2	0	-0.04	-0.07	-0.11	-0.15	-0.19	-0.23	-0.28	-0.32	-0.37	-0.42	-0.46	-0.51
Cost = 30	1	0.96	0.93	0.89	0.85	0.81	0.77	0.72	0.68	0.63	0.58	0.54	0.49
n = 10000 BEP (n)	10000	7274	6062	5385	4963	4683	4490	4353	4256	4185	4134	4096	4069
Price (R) = 50													
Revenue = 500000													
Fixed Cost = 200000 (FC)													
Variable Cost =300000 (VC) TVC (S_n)	$300,000	$163,676	$103,079	$69,244	$48,146	$34,138	$24,474	$17,644	$12,754	$9,218	$6,655	$4,795	$3,447
n R - FC	$300,000	$163,700	$103,100	$69,250	$48,150	$34,150	$24,500	$17,650	$12,800	$9,250	$6,700	$4,800	$3,450

Figure 2 graphically represents the effect of learning rates and the BEP. It is interesting to note that learning curve improves more quickly in the initial period compared to the later periods. Thus at a point of time it may not be feasible to improve learning rate further without involving too much additional resources in terms of time or cost. Further research is underway to utilize the effect of the change in BEP (due to learning) on other aspects of a business.

Figure 2
BEP Quantity at Learning Levels



This application of learning curves can be utilized in other areas. For example, in the example discussed above we have only accounted for variable costs. Fixed costs were not taken into consideration because they are fixed and not expected to change within the relevant range of production. However, through the use of activity based costing (ABC) it can be shown that many of the costs, thought to be fixed, vary with specific cost drivers. Learning curves also bring about qualitative improvements in addition to reducing the time required to complete a task. Assuming the company is using Total Quality Management, the improvement in learning rate will result in less defective parts, less inventory cost, and 'supply chain' economics. Similarly, many other applications in pricing strategy can be developed and empirical evidence provided.

Conclusion

We have demonstrated that recognition of learning curves in computing the BEP can have significant effect on the cost and profitability of a firm. In general, as the learning curves improve the variable cost per unit decreases and the contribution margin per unit increases. Increase in contribution margin lowers the BEP and consequently increases the profitability of the firm. Figure 2 shows that as the learning improves the break-even point may change. This has implications for setting budgeted targets, production runs, pricing, and financing arrangements. The information will be a very useful tool for the investors to understand that the profitability will increase significantly as the learning increases. Thus learning rates can be effectively utilized by an organization for planning purposes.

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